UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2002/2003

Februari/Mac 2003

JEE 543 – PEMPROSESAN ISYARAT DIGIT

Masa : 3 jam

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat berserta Lampiran (4 mukasurat) bercetak dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan ini.

Jawab **LIMA (5)** soalan.

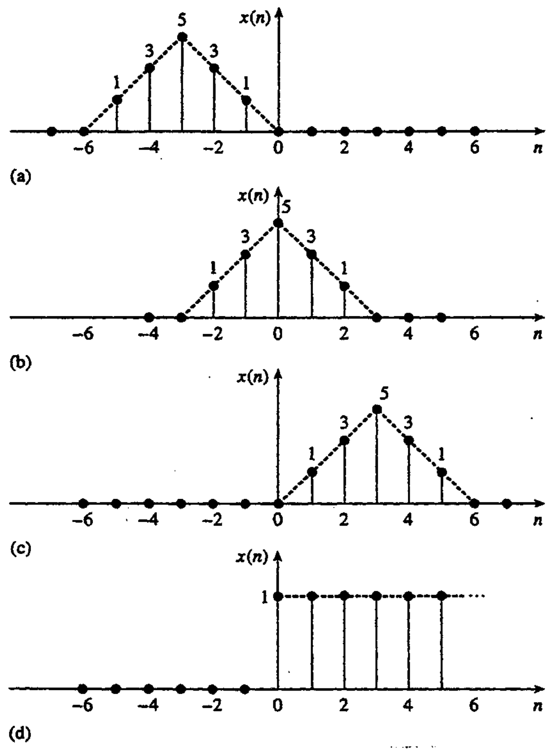
Agihan markah bagi soalan diberikan disut sebelah kanan soalan berkenaan.

Jawab semua soalan di dalam Bahasa Malaysia.

1. (a) Tentukan jelmaan-z bagi setiap jujukan masa diskrit di dalam Rajah 1.

Find the z-transform for each of the discrete-time sequences given in Figure 1.

(50%)



Rajah 1
Figure 1

(b) Dapatkan isyarat masa diskrit, x(n) yang dinyatakan oleh jelmaan-z berikut menggunakan kaedah perkembangan pecahan separuh.

Find the discrete-time signal, x(n), represented by the following z-transform using the partial fraction exparsion method.

$$x(z) = \frac{1}{(1-z^{-1})(1+0.8z^{-1})}$$

(50%)

2. (a) Pertimbangkan jujukan berikut:
Consider the following sequence:

$$(f[n]) = \{1, 1, 1, 1, 0, 0, 0, 0, 0, 0,\}$$

di mana N=10, dan anggapkan ianya adalah berkala.
where N=10, and is assumed periodic.

Dapatkan jelmaan Fourier diskrit untuk jujukan tersebut.
Find the discrete Fourier transform of teat sequence.

(50%)

(b) Dapatkan gambarajah rama-rama dari matriks berikut. (Gunakan kaedah pemusnahan dalam masa).

Find the butterflies diagram from the following matrix. (Using decimation-in-time method).

$$[\hat{W}_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w^{-1} & w^{-2} & w^{-3} \\ 1 & w^{-2} & w^{-4} & w^{-6} \\ 1 & w^{-3} & w^{-6} & w^{-9} \end{bmatrix}$$

[Hint : $w^{-1} = \exp\left(-j\frac{2\pi}{N}\right)$]

(50%)

3. Pertimbangkan penuras anjakan-tak-berbeza kausal lelurus dengan sistem fungsi.

Consider the causal linear shift-invariant filter with system function.

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})}$$

Lakarkan graf aliran isyarat untuk sistem ini menggunakan

Draw a signal flowgraph for this system using

(a) Bentuk terus I

Direct form I (30%)

(b) Bentuk terus II

Direct form II (30%)

(c) Satu kaskad bagi sistem peringkat pertama dan kedua dalam bentuk terus II.

A cascade of first and second-order systems realized in direct form II.

(40%)

4. (a) Dengan menganggap satu pendaraban kompleks memerlukan 1μs dan jumlah masa untuk mengira DFT ditentukan oleh jumlah masa yang diambil untuk menjalankan kesemua pendaraban.

Assume that a complex multiply takes 1μs and that the amount of time to compute a DFT is determined by the amount of time it takes to perform all of the multiplication.

(i) Berapakah masa yang diambil untuk mengira 1024 titik DFT secara terus.
How much time does it take to compute a 1024-point DFT directly?

(ii) Berapakah masa yang diperlukan jika FFT digunakan.
How much time is required if an FFT is used.

(iii) Ulangi bahagian (i) dan (ii) untuk 4096-titik DFT.
Repeat part (i) dan (ii) for 4096-point DFT.

(50%)

(b) Pertimbangkan jujukan panjang-terhad.
Consider the finite-length sequence.

$$x(u) = \delta(n) + 2\delta(n-5)$$

(i) Dapatkan jelmaan Fourier diskrit 10-titik untuk x(n).
Find the 10-point discrete Fourier transform of x(n).

(ii) Dapatkan jujukan yang mempunyai satu jelmaan Fourier Diskrit.
Find the sequence that has a discrete Fourier transform.

$$Y(k) = e^{j2k\frac{2\pi}{10}} X(k)$$

di mana X(k) adalah DFT 10-titik bagi x(n).
where X(k) is the 10-point DFT of x(n).

(50%)

5. Fungsi pindah berikut menunjukkan dua penuras yang berbeza yang memenuhi spesifikasi sambutan amplitud-frekuensi.

The following transfer functions represent two different filters meeting identical amplitude-frequency response specifications:

(i) $$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

di mana
where

b_0 = 0.498 181 9
b_1 = 0.927 477 7
b_2 = 0.498 181 9
a_1 = - 0.674 487 8
a_2 = - 0.363 348 2

(ii) $$H(z) = \sum_{k=o}^{11} h(k) z^{-k}$$

di mana
where

$h(0) = 0.546\ 032\ 80 \times 10^{-2} = h(11)$
$h(1) = -0.450\ 687\ 50 \times 10^{-1} = h(10)$
$h(2) = 0.691\ 694\ 20 \times 10^{-1} = h(9)$
$h(3) = -0.553\ 843\ 70 \times 10^{-1} = h(8)$
$h(4) = -0.634\ 284\ 10 \times 10^{-1} = h(7)$
$h(5) = 0.578\ 924\ 00 \times 10^{0} = h(6)$

Untuk setiap penuras:
For each filter:

(a) Nyatakan sama ada ianya penuras FIR atau IIR.
State whether it is an FIR or IIR filter.

(20%)

(b) Tunjukkan operasi penurasan dalam bentuk gambarajah blok dan tuliskan persamaan perbezaan.
Represent the filtering operation in a block diagram form and write down the difference equation, and

(50%)

(c) Tentukan dan berikan komen anda ke atas keperluan pengiraan dan penyimpanan.
Determine and comment on the computational and storage requirements.

(30%)

6. (a) Nyatakan langkah-langkah yang terlibat dalam merekabentuk satu penuras digit.
State the steps involves in designing a digital filter.

(30%)

(b) Satu penuras laluan jalur akan direkabentuk untuk memenuhi spesifikasi sambutan frekuensi berikut:

A FIR bandpass filter is to be designed to meet the following frequency response specifications:

Laluan jalur *Passband*	0.18 – 0.33 (normalized)
Lebar peralihan *Transition width*	0.04 (normalized)
Sisihan jalur henti *Stopband deviation*	0.001
Sisihan laluan jalur *Passband deviation*	0.05

(i) Lakarkan skim tolerensi untuk penuras ini.
Sketch the tolerance scheme for the filter.

(ii) Nyatakan frekuensi tepi jalur penuras dalam unit kilohertz, anggapkan frekuensi persampelan 10kHz, dan sisihan jalur henti dan laluan jalur dalam decibel.

Express the filter bandedge frequencies in the standard unit of kilohertz, assuming a sampling frequency of 10kHz, and the stopband and passband deviations in decibels.

(70%)

oooOooo

Property	Fourier Transform $x(t) \xleftrightarrow{FT} X(j\omega)$ $y(t) \xleftrightarrow{FT} Y(j\omega)$	Fourier Series $x(t) \xleftrightarrow{FS;\,\omega_o} X[k]$ $y(t) \xleftrightarrow{FS;\,\omega_o} Y[k]$ Period $= T$
Linearity	$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$	$ax(t) + by(t) \xleftrightarrow{FS;\,\omega_o} aX[k] + bY[k]$
Time shift	$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o}X(j\omega)$	$x(t - t_o) \xleftrightarrow{FS;\,\omega_o} e^{-jk\omega_o t_o}X[k]$
Frequency shift	$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$	$e^{jk_o\omega_o t}x(t) \xleftrightarrow{FS;\,\omega_o} X[k - k_o]$
Scaling	$x(at) \xleftrightarrow{FT} \frac{1}{\lvert a\rvert} X\left(\frac{j\omega}{a}\right)$	$x(at) \xleftrightarrow{FS;\,a\omega_o} X[k]$
Differentiation-time	$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$	$\frac{d}{dt}x(t) \xleftrightarrow{FS;\,\omega_o} jk\omega_o X[k]$
Differentiation-frequency	$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$	—
Integration/ Summation	$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$	—
Convolution	$\int_{-\infty}^{\infty} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FT} X(j\omega)Y(j\omega)$	$\int_{\langle T\rangle} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FS;\,\omega_o} TX[k]Y[k]$
Modulation	$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\nu)Y(j(\omega - \nu))\,d\nu$	$x(t)y(t) \xleftrightarrow{FS;\,\omega_o} \sum_{l=-\infty}^{\infty} X[l]Y[k - l]$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	$\frac{1}{T}\int_{\langle T\rangle} \lvert x(t)\rvert^2\,dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$
Duality	$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$
Symmetry	$x(t)$ real $\xleftrightarrow{FT} X^*(j\omega) = X(-j\omega)$ $x(t)$ imaginary $\xleftrightarrow{FT} X^*(j\omega) = -X(-j\omega)$ $x(t)$ real and even $\xleftrightarrow{FT} \mathrm{Im}\{X(j\omega)\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FT} \mathrm{Re}\{X(j\omega)\} = 0$	$x(t)$ real $\xleftrightarrow{FS;\,\omega_o} X^*[k] = X[-k]$ $x(t)$ imaginary $\xleftrightarrow{FS;\,\omega_o} X^*[k] = -X[-k]$ $x(t)$ real and even $\xleftrightarrow{FS;\,\omega_o} \mathrm{Im}\{X[k]\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FS;\,\omega_o} \mathrm{Re}\{X[k]\} = 0$

Discrete-Time FT	Discrete-Time FS
$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$	$x[n] \xleftrightarrow{DTFS;\,\Omega_o} X[k]$ $y[n] \xleftrightarrow{DTFS;\,\Omega_o} Y[k]$ Period $= N$
$ax[n] + by[n] \xleftrightarrow{DTFT} aX(e^{j\Omega}) + bY(e^{j\Omega})$	$ax[n] + by[n] \xleftrightarrow{DTFS;\,\Omega_o} aX[k] + bY[k]$
$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o}X(e^{j\Omega})$	$x[n - n_o] \xleftrightarrow{DTFS;\,\Omega_o} e^{-jk\Omega_o n_o}X[k]$
$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega-\Gamma)})$	$e^{jk_o\Omega_o n}x[n] \xleftrightarrow{DTFS;\,\Omega_o} X[k - k_o]$
$x_z[n] = 0, \quad n \neq lp$ $x_z[pn] \xleftrightarrow{DTFT} X_z(e^{j\Omega/p})$	$x_z[n] = 0, \quad n \neq lp$ $x_z[pn] \xleftrightarrow{DTFS;\,p\Omega_o} pX_z[k]$
—	—
$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega} X(e^{j\Omega})$	—
$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DTFT} \frac{X(e^{j\Omega})}{1 - e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$	—
$\sum_{l=-\infty}^{\infty} x[l]y[n - l] \xleftrightarrow{DTFT} X(e^{j\Omega})Y(e^{j\Omega})$	$\sum_{l=\langle N\rangle} x[l]y[n - l] \xleftrightarrow{DTFS;\,\Omega_o} NX[k]Y[k]$
$x[n]y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\Gamma})Y(e^{j(\Omega-\Gamma)})\,d\Gamma$	$x[n]y[n] \xleftrightarrow{DTFS;\,\Omega_o} \sum_{l=\langle N\rangle} X[l]Y[k - l]$
$\sum_{n=-\infty}^{\infty} \|x[n]\|^2 = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} \|X(e^{j\Omega})\|^2\,d\Omega$	$\frac{1}{N}\sum_{n=\langle N\rangle} \|x[n]\|^2 = \sum_{k=\langle N\rangle} \|X[k]\|^2$
$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$	$X[n] \xleftrightarrow{DTFS;\,\Omega_o} \frac{1}{N}x[-k]$
$x[n]$ real $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = X(e^{-j\Omega})$ $x[n]$ imaginary $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = -X(e^{-j\Omega})$ $x[n]$ real and even $\xleftrightarrow{DTFT} \mathrm{Im}\{X(e^{j\Omega})\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFT} \mathrm{Re}\{X(e^{j\Omega})\} = 0$	$x[n]$ real $\xleftrightarrow{DTFS;\,\Omega_o} X^*[k] = X[-k]$ $x[n]$ imaginary $\xleftrightarrow{DTFS;\,\Omega_o} X^*[k] = -X[-k]$ $x[n]$ real and even $\xleftrightarrow{DTFS;\,\Omega_o} \mathrm{Im}\{X[k]\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFS;\,\Omega_o} \mathrm{Re}\{X[k]\} = 0$

E.1 *Basic z-Transforms*

Signal	*Transform*	*ROC*
$\delta[n]$	1	All z
$u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$\alpha^n u[n]$	$\dfrac{1}{1 - \alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n\alpha^n u[n]$	$\dfrac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$[\cos(\Omega_1 n)]u[n]$	$\dfrac{1 - z^{-1}\cos\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z\rvert > 1$
$[\sin(\Omega_1 n)]u[n]$	$\dfrac{z^{-1}\sin\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z\rvert > 1$
$[r^n\cos(\Omega_1 n)]u[n]$	$\dfrac{1 - z^{-1}r\cos\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2z^{-2}}$	$\lvert z\rvert > r$
$[r^n\sin(\Omega_1 n)]u[n]$	$\dfrac{z^{-1}r\sin\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2z^{-2}}$	$\lvert z\rvert > r$

BILATERAL TRANSFORMS FOR SIGNALS THAT ARE NONZERO FOR $n < 0$

Signal	*Bilateral Transform*	*ROC*
$u[-n-1]$	$\frac{1}{1 - z^{-1}}$	$\|z\| < 1$
$-\alpha^n u[-n-1]$	$\frac{1}{1 - \alpha z^{-1}}$	$\|z\| < \|\alpha\|$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$

E.2 *z-Transform Properties*

Signal	*Unilateral Transform*	*Bilateral Transform*	*ROC*
$x[n]$	$X(z)$	$X(z)$	R_x
$y[n]$	$Y(z)$	$Y(z)$	R_y
$ax[n] + by[n]$	$aX(z) + bY(z)$	$aX(z) + bY(z)$	At least $R_x \cap R_y$
$x[n-k]$	See below	$z^{-k}X(z)$	R_x except possibly $\|z\| = 0, \infty$
$\alpha^n x[n]$	$X\left(\frac{z}{\alpha}\right)$	$X\left(\frac{z}{\alpha}\right)$	$\|\alpha\| R_x$
$x[-n]$	—	$X\left(\frac{1}{z}\right)$	$\frac{1}{R_x}$
$x[n] * y[n]$	$X(z)Y(z)$	$X(z)Y(z)$	At least $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	$-z\frac{d}{dz}X(z)$	R_x except possibly addition or deletion of $z = 0$